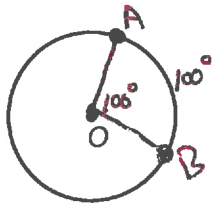
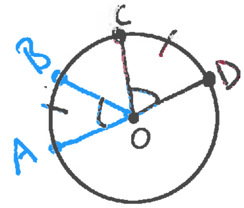
Geometry R – Mr. Bo
Unit 10 – Day 2 Notes

Name: notes.
Date:______________

Central & Inscribed Angles

Central Angle – An angle contained in a circle with its vertex at the circle's center.





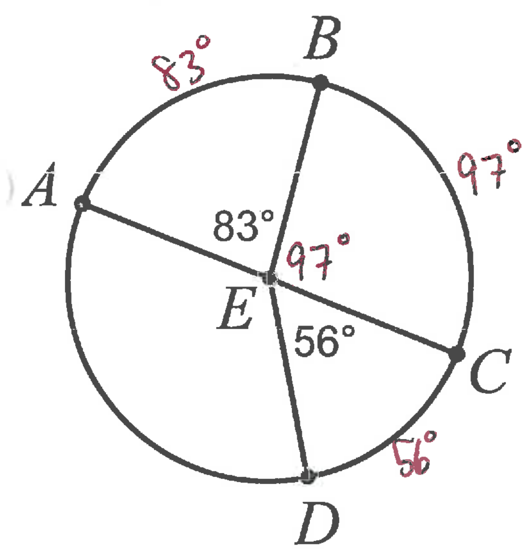
if $\angle AOB \cong \angle COD$
then
$\overset{\frown}{AB} \cong \overset{\frown}{CD}$

Theorem – The measure of a central angle is equal to the measure of the arc cut.

Theorem – In a circle or ≅ circles, ≅ central angles cut ≅ arcs.

1. Apply the Central angle theorems to complete the problems.

Circle E with Diameter AC.



Find:

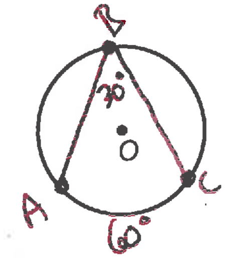
a. $m\overset{\frown}{AB} = m\angle AEB = 83°$

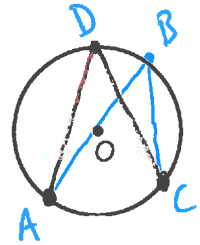
b. $m\overset{\frown}{BC} = 180 - 83° = 97°$

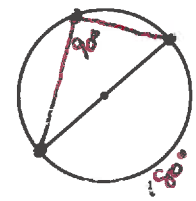
c. $m\overset{\frown}{ABD} = 83 + 97 + 56 = 236°$

d. $m\angle AED = 180° - 56° = 124°$

Inscribed Angles – An angle contained in a circle with its vertex on the circle.







Theorem – The measure of an inscribed angle is ½ the measure of the arc cut.

$m\angle ABC = \frac{1}{2} m\overset{\frown}{AC}$

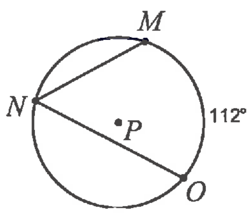
Theorem – In a circle, two inscribed angles that cut the same arc are congruent

$\angle ADC \cong \angle ABC$

Theorem – An angle inscribed in a semi-circle is 90°.

2. Apply the inscribed angle theorems to complete each problem.

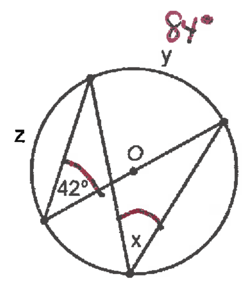
a. Circle P. Find $m\angle MNO$.



$m\angle MNO = \frac{1}{2}(112)$

$= 56°$

b. Find x, y, and z for circle O.



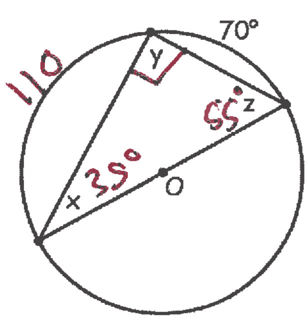
$y = 2(42) = 84°$

$x = 42°$

$z = 180 - 84$

$= 96°$

c. Find x, y, and z for circle O.



$y = 90°$

$x = \frac{1}{2}(70)$

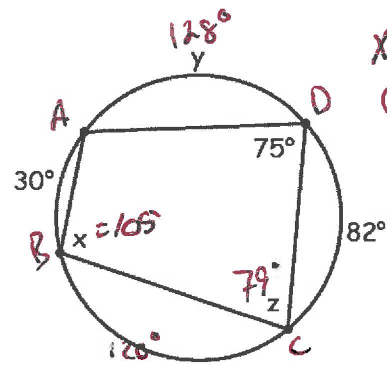
$= 35°$

$z = \frac{1}{2}(110)$

$= 55°$

d. Find x, y, and z for circle O.

(***A Helpful but not necessary Theorem*** *– The opposite angles of an inscribed quadrilateral are supplementary.)*



$x + 75 = 180$

$x = 105°$

$y = 360 - (30 + 120 + 82)$

$= 360 - 232$

$= 128°$

$z = \frac{1}{2}(30 + 128)$

$= \frac{1}{2}(158)$

$= 79°$

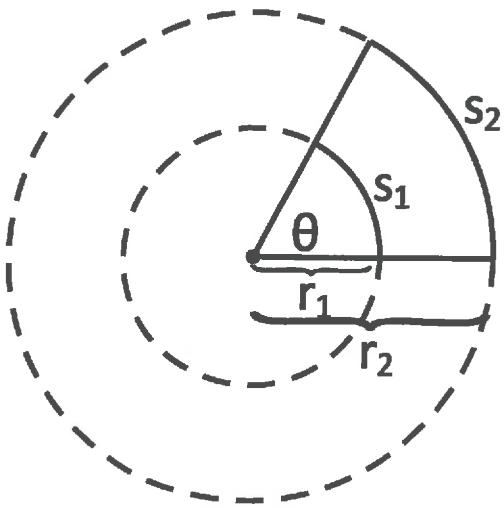
$m\overset{\frown}{ABC} = 2(75) = 150°$

$m\overset{\frown}{BC} = 150 - 30$

$= 120°$

Similarity of Circles & Radian Measure:

Theorem – All circles are similar to each other.



3a. Since the large circle and the small circle are similar, the following proportion compares their corresponding parts:

$$\frac{r_1}{r_2} = \frac{s_1}{s_2}$$

b. Switching the extremes of the proportion produces:

$$\frac{s_2}{r_2} = \frac{s_1}{r_1}$$

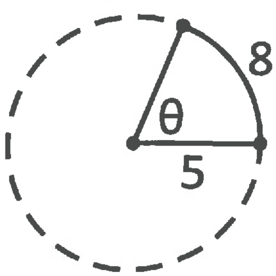
no matter how big the circle, the ratio of s to r is always the same for the angle θ.

The **Radian Measure** of a central angle is the ratio of the length of the arc cut by the angle to the radius of the circle. In other words, in each circle above, the following equation gives the radian measure angle θ (*the Greek letter Theta*):

$$\theta = \frac{s}{r}$$

3.

a. Find θ, to the nearest tenth of a radian.

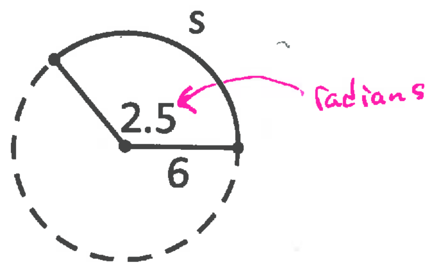


$\theta = \frac{s}{r}$

$\theta = \frac{8}{5}$

$\theta = 1.6$ radians.

b. Find s, to the nearest tenth.



$\theta = \frac{s}{r}$

$2.5 = \frac{s}{6}$

$s = 6(2.5)$

$s = 15$ units

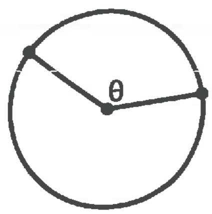
semi circle

Converting Radians to Degrees: To convert a radian measure to degrees, multiply the angle by $\frac{180^\circ}{\pi}$.

half circumference of a circle

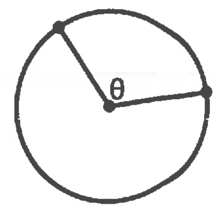
4. Convert each radian measure to degrees:

a. $\theta = \frac{3\pi}{4}$ radians



$\frac{3\pi}{4} \cdot \frac{180}{\pi} = \frac{3(\cancel{180})^{45}}{\cancel{4}_{1}} = 135^\circ$

b. $\theta = 2$ radians

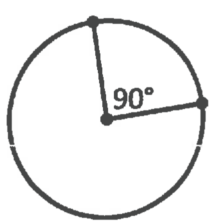


$\frac{2}{1} \cdot \frac{180}{\pi} = \frac{360}{\pi} = 114.6^\circ$

Converting Degrees to Radians: To convert a degree measure to radians, multiply by $\frac{\pi}{180^0}$.

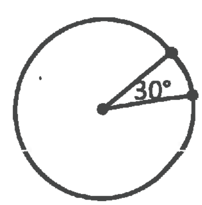
5. Convert each degree measure to radians (give answers in terms of π)

a.



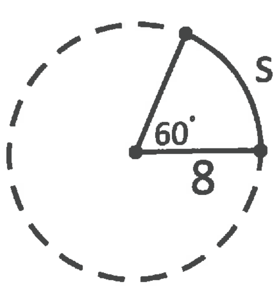
$\frac{90^\circ}{1} \cdot \frac{\pi}{180^\circ} = \frac{90\pi}{180} = \frac{\pi}{2}$ radians

b.



$\frac{30^\circ}{1} \cdot \frac{\pi}{180^\circ} = \frac{30\pi}{180} = \frac{\pi}{6}$ radians.

6. Find s, to the nearest tenth.



* must convert to radians First!

$\theta = \frac{60^\circ}{1} \cdot \frac{\pi}{180^\circ} = \frac{60\pi}{180} = \frac{\pi}{3}$ radians.

$\theta = \frac{s}{r}$

$\frac{\pi}{3} = \frac{s}{8}$

$s = \frac{8\pi}{3} \approx 8.4$ units.